QUIZ 5 - CALCULUS 2 (2020/12/31)

1. (8 pts) Find all values of k so that the given y is a solution to the given differential equation.

   (a) (4 pts) $y = k, \quad \dfrac{dy}{dx} = \dfrac{y(1+y)(4-y)}{1+y^2}$.

   Solution:

$$0 = \frac{k(1+k)(4-k)}{1+k^2} \quad \Rightarrow \quad k = 0, -1, 4$$

□

   Grading: Plug the function in the differential equation (1 pt). Correct answer (3 pts).

   (b) (4 pts) $y = x^k, \quad y'' + \dfrac{y'}{x} - \dfrac{4y}{x^2} = 0$.

   Solution:

$$k(k-1)x^{k-2} + \frac{kx^{k-1}}{x} - \frac{4x^k}{x^2} = 0 \quad \Rightarrow \quad k^2 - k + k - 4 = 0 \Rightarrow k = \pm 2.$$

□

   Grading: Plug the function in the differential equation (2 pts). Correct answer (2 pts).

2. (12 pts) Solve the given initial value problem.

   (a) (6 pts) $\dfrac{dy}{dx} = \dfrac{y^2}{1+x^2}, \quad y(0) = \dfrac{1}{3}$.

   Solution:

$$\int \frac{1}{y^2}\, dy = \int \frac{1}{1+x^2}\, dx$$

$$\frac{-1}{y} = \tan^{-1} x + C, \quad y(0) = \frac{1}{3}, \quad -3 = 0 + C, \ C = -3$$

$$y = \frac{-1}{\tan^{-1} x - 3}$$

□

   Grading: Separating (2 pts). Integrals (3 pts). Finding the value of C (1 pt). Final answer do not need to be simplified to $y = f(x)$ form.

   (b) (6 pts) $\dfrac{dy}{dx} - \dfrac{2xy}{x^2+1} = 4x, \quad y(0) = 5$.

   Solution:

$$P(x) = \frac{-2x}{1+x^2}, \quad Q(x) = 4x$$

   The integrating factor $I(x)$ is

$$I(x) = e^{\int P(x)\ dx}, \quad I(x) = e^{-\ln(1+x^2)} = \frac{1}{(1+x^2)}$$

$$\frac{dy}{dx} \cdot I(x) - \frac{2xy \cdot I(x)}{x^2+1} = \frac{d}{dx}(I(x) \cdot y) = 4x \cdot I(x)$$

$$\frac{y}{1+x^2} = \int \frac{4x}{1+x^2}\, dx = 2\ln(1+x^2) + C$$

$$y(0) = 5, \quad 5 = 0 + C, \ C = 5$$

$$y = 2(1 + x^2)\ln(1 + x^2) + 5(1 + x^2)$$

□

Grading: Integrating factor (2 pts). Integrals (3 pts). Finding the value of C (1 pt). Final answer do not need to be simplified to $y = f(x)$ form.